

A NOTE ON MITSUMATSU'S CONSTRUCTION OF A LEAFWISE SYMPLECTIC FOLIATION

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ABSTRACT. Mitsumatsu[9] constructed a leafwise symplectic structure of the Lawson foliation on S^5 . Combining his construction with a previous result of the author[12], we obtain a path of almost contact structures connecting a contact structure to a leafwise symplectic foliation on a certain closed manifold of dimension > 3 . This leads us to generalize the notion of confoliation (Eliashberg-Thurston[1]) to higher dimension by means of almost contact geometry.

1. INTRODUCTION

Let \mathcal{F} be a codimension one foliation on an oriented $(2n+1)$ -manifold M^{2n+1} defined by a non-singular 1-form α with $\alpha \wedge d\alpha = 0$. Suppose that there exists a 2-form ω on M^{2n+1} satisfying $\alpha \wedge \omega^n > 0$ and $\alpha \wedge d\omega = 0$. Then $\omega|_{\ker \alpha}$ is called a leafwise symplectic structure of \mathcal{F} . Recently Mitsumatsu[9], motivated by an effort of Verjovsky et al., constructed a leafwise symplectic structure of the Lawson foliation on S^5 . The Lawson foliation is spinnable, i.e., arises naturally from a certain open-book decomposition. The main point of Mitsumatsu's construction is a standard cohomology calculation which guarantees that a certain 2-form on the binding extends to a page-wise closed 2-form. We can generalize his result to a class of spinnable foliations characterized by a similar extension property. However, at present, we have no examples of such foliations essentially different from the Lawson foliation (see §4 for further discussion).

In order to explain our motivation and to state our result, we recall almost contact geometry. An almost contact structure is a G -structure on M^{2n+1} by the subgroup $G \subset GL^+(2n+1, \mathbb{R})$ consisting of 1-jets of local contactomorphisms of the 1-jet space $(J^1(\mathbb{R}^n, \mathbb{R}), 0)$. An almost contact structure corresponds bijectively to a pair $([\alpha], [\omega])$ of conformal classes of 1- and 2-forms which satisfies $[\alpha] \wedge [\omega]^n > 0$ (see for example Ogiue[13]). Note that Sasaki's (ϕ, ξ, η) -structure further requires a fixed representative $\omega \in [\omega]$, a fixed almost complex structure J on $\ker[\alpha]$ compatible with $\omega|_{\ker[\alpha]}$, and a fixed positive section ξ of the missing (=degenerate) direction of ω . Then we put $\phi|_{\ker[\alpha]} = J$, $\phi(\xi) = 0$, $\eta \in [\alpha]$, and $\eta(\xi) = 1$ to recover the setting in Sasaki[14]. Note also that, if the structure group of TM^{2n+1} admits a reduction to $U(n)$, M^{2n+1} clearly admits an almost contact structure. (Gray first called such a reduction an almost contact structure.)

Leafwise symplectic foliations and (positive) contact forms clearly yield examples of almost contact structures. An almost contact structure on a 3-manifold is nothing but fixing a co-oriented and oriented plane field. Eliashberg and Thurston considered a path of plane fields connecting a 3-dimensional contact structure to a codimension one foliation. Here they required that the path stays in the closure $\bar{\mathcal{C}}$ of the space \mathcal{C} of contact structures. A plane field in $\bar{\mathcal{C}} \cup \mathcal{I}$ is called a confoliation (contact+foliation) where \mathcal{I} denotes the space of completely integrable plane fields.¹ An important phenomena in confoliation theory is a convergence of contact structures to a foliation. The author[11] showed that every contact structure (isotopically) deforms into a limit foliation which is naturally associated to a supporting open-book decomposition (see also Etnyre[2]). Such a foliation is called a spinnable foliation. The convergence of contact structures to a spinnable foliation is surprisingly nice since there exists a representative in every transverse

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¹Though a (co-oriented) confoliation is formally defined in [1] as a plane field $\ker \alpha$ such that $\alpha \wedge d\alpha \geq 0$, we may restrict ourselves, at least practically, to the case where it belongs to $\bar{\mathcal{C}} \cup \mathcal{I}$ throughout [1].

link-type which is also transverse to the limit foliation (see Mitsumatsu-M.[10]). This implies that the limit foliation satisfies the relative Thurston inequality if and only if the original contact structure is tight. The author[12] showed that we can also deform a certain higher dimensional contact structure into a spinnable foliation. In this article *we improve this result in the light of Mitsumatsu's construction. Namely we construct a path of almost contact structures $([\alpha_t], [\omega_t])$ on a certain closed oriented manifold M^{2n+1} of dimension $2n+1 > 3$ such that*

- $[\alpha_t]$ is a contact structure for $0 \leq t < 1$,
- initially $\omega_0 = d\alpha_0$ for $\exists \alpha_0 \in [\alpha_0]$ and $\exists \omega_0 \in [\omega_0]$, and
- $[\alpha_1]$ defines a foliation with leafwise symplectic structure $\omega_1|_{\ker[\alpha_1]}$ for $\exists \omega_1 \in [\omega_1]$.

This phenomena leads us to generalize the notion of confoliation to higher dimension: Let \mathcal{C} denote the set of almost contact structures arising from contact forms, and \mathcal{I} the set of those arising from leafwise almost symplectic foliations. Then a possible generalization of confoliation is an element of $\overline{\mathcal{C}} \cup \mathcal{I}$ in the topological space of all almost contact structures on M^{2n+1} .

Ibort and Martínez Torres began studying an almost contact structure $([\alpha], [\omega])$ such that $[\omega]$ is represented by a closed form ω . The closed 2-form ω is called a 2-calibration of $\ker[\alpha]$. Then they constructed a Donaldson type submanifold ([5]) and Lefschetz type pencil ([6]). If $[\alpha]$ defines a foliation \mathcal{F} , the 2-calibration ω clearly defines a leafwise symplectic structure of \mathcal{F} . A 2-calibrated foliation on a 3-manifold is nothing but a taut foliation, while every (co-oriented and oriented) foliation on a 3-manifold is leafwise symplectic. Let ω be a 2-calibration of a foliation \mathcal{F} on a closed manifold M^{2n+1} of dimension $2n+1 > 3$. Then Martínez Torres[8], using an idea of Seidel[15], showed that there exists a closed 3-dimensional transversal $N^3 \subset M$ of \mathcal{F} which inherits a taut foliation and meets every leaf of \mathcal{F} in a single leaf. Thus roughly speaking, a 2-calibrated foliation is a leafwise symplectic foliation which is obtained by “fattening” a 3-dimensional taut foliation. On the other hand the transverse structure of the spinnable foliation coincides with that of the Reeb foliation on S^3 . The author speculates that a leafwise symplectic foliation could inherit a low dimensional peculiarity in foliation theory, e.g. the Novikov closed leaf theorem.

2. STATEMENT OF THE RESULT

Let us start with the definition of a supporting open-book decomposition.

Definition 2.1 (Giroux[4]). Let $\ker \alpha$ be a positive contact structure on a closed oriented $(2n+1)$ -manifold M^{2n+1} ($\alpha \wedge d\alpha^n > 0$), and \mathcal{O} a positively oriented open-book decomposition of M^{2n+1} by pages $P_\theta \subset M^{2n+1}$ ($\theta \in \mathbb{R}/2\pi\mathbb{Z}$). Suppose that the oriented binding $N^{2n-1} = \partial P_0 = \partial P_\theta$ inherits a positive contact structure, and there exists a representative $\alpha_0 \in [\alpha]$ such that

$$d\theta \wedge \{d\alpha_0\}^n > 0 \quad \text{on} \quad M^{2n+1} \setminus N^{2n-1}.$$

Then we say that \mathcal{O} supports the contact structure $\ker \alpha$, and α_0 is an adapted contact form.

Giroux[4] showed that every contact structure on M^{2n+1} can be supported by an open-book decomposition. The open-book decomposition together with the symplectic data of the page and the monodromy recovers the contact structure uniquely up to isotopy (and thus “supports” it). This fact is at least partially recognized by Thurston and Winkelnkemper as early as '75.

Take a neighbourhood $N^{2n-1} \times D^2$ of N^{2n-1} and polar coordinates (r, θ) of D^2 compatible with the fibration $\{P_\theta\}$. Using the Gray stability we can isotopically deform $\ker \alpha$ to be axisymmetric on $N^{2n-1} \times D^2$. Thus we may assume that an adapted contact form α_0 satisfies

$$\alpha_0|(N^{2n-1} \times D^2) = f_0(r)\mu + g_0(r)d\theta$$

where

- μ is any contact form on N^{2n-1} defining a contact structure isotopic to $\ker \alpha \cap TN^{2n-1}$,
- $f_0(r)$ is a positive function of r on $N^{2n-1} \times D^2$ with $f'_0(r) < 0$ on $(0, 1]$, and
- $g_0(r)$ is a weakly increasing function with $g_0(r) \equiv r^2$ on $[0, 1/2]$ and $g_0(r) \equiv 1$ near $r = 1$.

Then we can prove the following convergence theorem (see §3 for the proof).

Theorem 2.2 ([12]). *In the above setting, suppose moreover that the binding N^{2n-1} admits a non-zero closed 1-form ν with $\nu \wedge (d\mu)^{n-1} \equiv 0$, i.e., the Reeb field of μ has the local first integral defined by ν ($n > 1$). For functions $f_1(r)$, $g_1(r)$, $h(r)$ and $e(r)$ on $[0, 1]$ such that*

- $f_1 \equiv 1$ on $[0, 1/4]$, $f_1 \equiv 0$ on $[1/2, 1]$, $f_1' < 0$ on $(1/4, 1/2)$,
- $g_1 \equiv 1$ on $[3/4, 1]$, $g_1 \equiv 0$ on $[0, 1/2]$, $g_1' > 0$ on $(1/2, 3/4)$,
- $h \equiv 1$ on $[0, 3/4]$, $h \equiv 0$ near $r = 1$,
- e is supported near $r = 1/2$, and $e(1/2) \neq 0$ (see Figure 1),

we put $f_t(r) = (1-t)f_0(r) + tf_1(r)$, $g_t(r) = (1-t)g_0(r) + tg_1(r)$, $\tau = (1-t)^2$, and

$$\alpha_t = \begin{cases} f_t(r)\{(1-t)\mu + th(r)\nu\} + g_t(r)d\theta + te(r)dr & \text{on } N^{2n-1} \times D^2 \\ \tau\alpha_0 + (1-\tau)d\theta & \text{on } M^{2n+1} \setminus (N^{2n-1} \times D^2). \end{cases}$$

Then $\{\ker \alpha_t\}_{0 \leq t < 1}$ is a family of contact structures convergent to the foliation defined by α_1 .

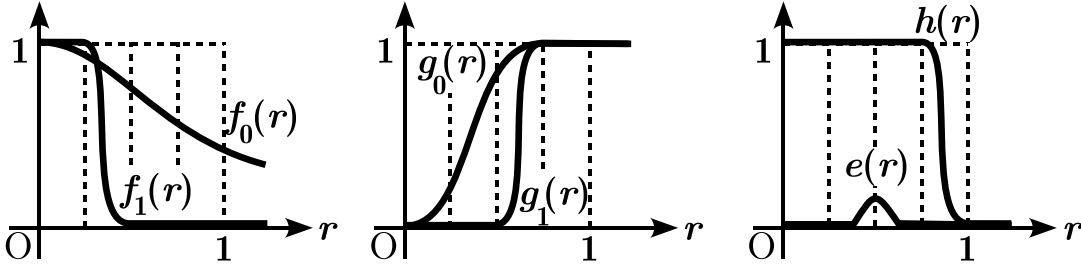


FIGURE 1. The functions.

The result of this article is the following improvement of Theorem 2.2 (see §3 for the proof).

Theorem 2.3. *In Theorem 2.2, further assume that there exists a closed 2-form Ω on N^{2n-1} such that*

- (1) $\nu \wedge (\varepsilon\Omega + d\mu)^{n-1} > 0$ for a sufficiently small positive constant ε , and
- (2) Ω extends to a pagewise closed 2-form on M^{2n+1} .

From the condition (2) we can take a 2-form $\tilde{\Omega}$ on M^{2n+1} such that

- (2-1) $d\theta \wedge d\tilde{\Omega} = 0$ holds on $M^{2n+1} \setminus N^{2n-1}$, and
- (2-2) $\tilde{\Omega}|_{N^{2n-1} \times D^2} = \Omega + e(r)d\theta \wedge \nu$.

Then taking smaller $\varepsilon > 0$ if necessary, the 2-form $\omega_t = d\alpha_0 + t\varepsilon\tilde{\Omega}$ satisfies

- (i) $\omega_0 = d\alpha_0$,
- (ii) $\alpha_1 \wedge d\omega_1 \equiv 0$, and
- (iii) $\alpha_t \wedge \omega_t^n > 0$ ($0 \leq t \leq 1$).

Namely, we can deform the almost contact structure $([\alpha_0], [d\alpha_0])$ of the adapted contact form α_0 , keeping the first component contact, into that of the limit leafwise symplectic foliation.

Example 2.4. We obtain the Lawson foliation on S^5 from Theorem 2.3 in the case where

- $\ker \alpha$ is the standard contact structure on the unit hypersphere $S^5 \subset \mathbb{C}^3 \ni (X, Y, Z)$,
- \mathcal{O} is the Milnor fibration $\arg(X^3 + Y^3 + Z^3)|_{S^5}$ around $N^3 = \{X^3 + Y^3 + Z^3 = 0\} \cap S^5$,
- ν is the pull-back of the length 1-form on S^1 under the composition $p_2 \circ p_1 : N^3 \rightarrow T^2 \rightarrow S^1$ of the restriction $p_1 = p|_{N^3}$ of the Hopf fibration $p : \mathbb{C}^3 \rightarrow \mathbb{C}P^2$ with an arbitrary trivial circle bundle $p_2 : \pi_1(N^3) \approx T^2 \rightarrow S^1$, and
- Ω is a leafwise area form of the T^2 -foliation on N^3 defined by integrating $\ker \nu$.

Indeed Mitsumatsu[9] showed that Ω extends to a pagewise closed 2-form on S^5 . The author[12] used the same object as an example for Theorem 2.2.

3. PROOFS

Proof of Theorem 2.2. We review the calculation in [12] for reader's convenience. We omit the calculation on $M^{2n+1} \setminus (N^{2n-1} \times D^2)$, where $\{\alpha_t\}$ is just a line parametrized by τ . We have

$$\begin{aligned}\alpha_t &= (1-t)f_t(r)\mu + tf_t(r)h(r)\nu + g_t(r)d\theta + te(r)dr \\ d\alpha_t &= (1-t)f'_t dr \wedge \mu + t\{f_t h\}' dr \wedge \nu + g'_t dr \wedge d\theta \\ \alpha_t \wedge (d\alpha_t)^n &= n(1-t)\frac{g'_t f_t - f'_t g_t}{r}\mu \wedge (d\mu)^{n-1} \wedge (rdr \wedge d\theta) > 0 \quad (0 \leq t < 1).\end{aligned}$$

On the other hand we have

$$\begin{aligned}\alpha_1 &= f_1(r)\nu + g_1(r)d\theta + e(r)dr \neq 0 \\ d\alpha_1 &= f'_1 dr \wedge \nu + g'_1 dr \wedge d\theta \\ \alpha_1 \wedge d\alpha_1 &= \frac{g'_1 f_1 - f'_1 g_1}{r}\nu \wedge (rdr \wedge d\theta) \equiv 0.\end{aligned}$$

Note that the orientation of the closed leaf $\{r = 1/2\}$ depends on the sign of $e(1/2)$. \square

Proof of Theorem 2.3. (i) $\omega_0 = d\alpha_0 + 0 = d\alpha_0$.

$$(ii) \quad \alpha_1 \wedge d\tilde{\Omega} = \begin{cases} (f_1\nu + g_1d\theta + e dr) \wedge (\varepsilon e'(r)dr \wedge d\theta \wedge \nu) = 0 & \text{on } N^{2n-1} \times D^2 \\ d\theta \wedge d\tilde{\Omega} = 0 & \text{on } M^{2n+1} \setminus (N^{2n-1} \times D^2). \end{cases}$$

(iii) On $M^{2n+1} \setminus (N^{2n-1} \times D^2)$, we see from

$$\{\tau\alpha_0 + (1-\tau)d\theta\} \wedge (d\alpha_0)^n > 0$$

that $\alpha_t \wedge \omega_t^n > 0$ holds for sufficiently small $\varepsilon > 0$. Hereafter we restrict everything to $N \times D^2$. We use the four term expression

$$\alpha_t = (1-t)f_t(r)\mu + tf_t(r)h(r)\nu + g_t(r)d\theta + te(r)dr$$

and prove the non-negativity of $(\text{each term}) \wedge \omega_t^n$ individually.

Lemma 3.1. $(1-t)f_t(r)\mu \wedge \omega_t^n \geq 0$ ($0 < \varepsilon \ll 1$) with equality iff $(1-t)f_t(r)g'_0(r)/r = 0$.

Proof. The left hand side of the inequality is

$$\begin{aligned}(1-t)f_t(r)\mu \wedge \omega_t^n &= (1-t)f_t(r)\mu \wedge \{f_0(r)d\mu + f'_0(r)dr \wedge \mu + g'_0(r)dr \wedge d\theta + t\varepsilon\Omega + t\varepsilon e(r)d\theta \wedge \nu\}^n \\ &= n(1-t)f_t(r)g'_0(r)\mu \wedge dr \wedge d\theta \wedge \{f_0(r)d\mu + t\varepsilon\Omega\}^{n-1}.\end{aligned}$$

Since $rdr \wedge d\theta \wedge \mu \wedge d\mu^{n-1}$ is positive and $1/f_0(r)$ is bounded, we see that $(1-t)f_t(r)\mu \wedge \omega_t^n$ is positive on the subset $\{(1-t)f_t(r)g'_0(r)/r > 0\}$ for sufficiently small $\varepsilon > 0$. \square

Lemma 3.2. $tf_t(r)h(r)\nu \wedge \omega_t^n \geq 0$ ($0 < \varepsilon \ll 1$) with equality iff $tf_t(r)h(r)g'_0(r)/r = 0$.

Proof. The left hand side of the inequality is

$$\begin{aligned}tf_t(r)h(r)\nu \wedge \omega_t^n &= tf_t(r)h(r)\nu \wedge \{f_0(r)d\mu + f'_0(r)dr \wedge \mu + g'_0(r)dr \wedge d\theta + t\varepsilon\Omega + t\varepsilon e(r)d\theta \wedge \nu\}^n \\ &= nt f_t(r)h(r)g'_0(r)\nu \wedge dr \wedge d\theta \wedge \{f_0(r)d\mu + t\varepsilon\Omega\}^{n-1}.\end{aligned}$$

Since $rdr \wedge d\theta \wedge \nu \wedge (\varepsilon\Omega + d\mu)^{n-1}$ is positive for small $\varepsilon > 0$ and $1/f_0(r)$ is bounded, we see that $tf_t(r)h(r)\nu \wedge \omega_t^n$ is positive on the subset $\{tf_t(r)h(r)g'_0(r)/r > 0\}$ for sufficiently small $\varepsilon > 0$. \square

Lemma 3.3. $g_t(r)d\theta \wedge \omega_t^n \geq 0$ ($0 < \varepsilon \ll 1$) with equality iff $g_t(r)/r = 0$.

Proof. $g_t(r)d\theta \wedge \omega_t^n = ng_t(r)f'_0(r)d\theta \wedge dr \wedge \mu \wedge \{f_0(r)d\mu + t\varepsilon\Omega\}^{n-1}$. \square

Lemma 3.4. $te(r)dr \wedge \omega_t^n \geq 0$ ($0 < \varepsilon \ll 1$) with equality iff $te(r) = 0$.

Proof. $te(r)dr \wedge \omega_t^n = nt^2\{e(r)\}^2 dr \wedge d\theta \wedge \nu \wedge \{f_0(r)d\mu + t\varepsilon\Omega\}^{n-1}$. \square

To complete the proof of Theorem 2.3, let us prove that the equalities in the above lemmata do not hold simultaneously.

In the case where $0 \leq t < 1$, the equality of Lemma 3.3 holds only on $N^{2n-1} = \{r = 0\}$. On the other hand, the equality of Lemma 3.1 fails on N^{2n-1} . Thus the above lemmata imply that $\alpha_t \wedge \omega_t^n$ is strictly positive.

In the case where $t = 1$, the equality of Lemma 3.3 holds only on $\{r \leq 1/2\}$, that of Lemma 3.4 does not hold near $\{r = 1/2\}$, and that of Lemma 3.2 does not hold on $\{r < 1/2\}$. Thus the above lemmata imply that $\alpha_1 \wedge \omega_1^n$ is strictly positive. This completes the proof. \square

4. FURTHER DISCUSSION

The conditions (1) and (2) in Theorem 2.3 are considerably severe since it implies that we can perturb the symplectic structure on the page $P = P_0$ so that we can attach to it a cylindrical end with a periodic symplectic structure. (This end coils into a closed symplectic leaf $\{r = 1/2\}$; see Mitsumatsu[9].) While Theorem 2.2 does not concern the symplectic monodromy, Theorem 2.3 requires that the symplectic monodromy *further* preserves the other end-periodic non-exact symplectic structure than the original exact one. So it is a wonder that the page leaf and the monodromy of the Lawson foliation simultaneously do satisfy these conditions.

As is mentioned in §1, we put

$$\begin{aligned} \mathcal{C} &= \{([\alpha], [\omega]): \text{almost contact structure} \mid \ker[\alpha]: \text{contact structure}\} \quad \text{and} \\ \mathcal{I} &= \{([\alpha], [\omega]): \text{almost contact structure} \mid \ker[\alpha] = T\mathcal{F} \text{ for } \exists \text{foliation } \mathcal{F}\} \end{aligned}$$

and define a generalized confoliation as an element of $\overline{\mathcal{C}} \cup \mathcal{I}$ in the space of all almost contact structures. Then each element of \mathcal{I} is a leafwise almost symplectic foliation. In future \mathcal{I} might be replaced by the subset consisting of leafwise symplectic foliations. As is described below, there is a wide difference between leafwise almost symplecticity and leafwise symplecticity.

The existence of a leafwise symplectic structure is not a direct consequence of the convergence of contact structures to a foliation. Moreover, even if the deformation can be realized as a family of almost contact structures, the limit foliation does not always admit a leafwise symplectic structure. To see these facts, consider a linear perturbation $\theta_t = \theta_0 + tv_0$ of an 1-form. If $\ker \theta_0$ is completely integrable, i.e., if $d\theta_0 = \theta_0 \wedge \gamma$ holds for some γ , we have

$$\begin{aligned} \theta_t \wedge (d\theta_t)^n &= (\theta_0 + tv_0) \wedge (\theta_0 \wedge \gamma + tdv_0)^n \\ &= t^n \theta_0 \wedge (dv_0 + \gamma \wedge v_0)^n + O(t^{n+1}). \end{aligned}$$

Thus if $\omega := dv_0 + \gamma \wedge v_0$ satisfies $\theta_0 \wedge \omega^n > 0$, the perturbation θ_t is a contact form for $0 < t \ll 1$. Then $\omega|_{\ker \theta_0}$ is certainly non-degenerate, however it is not always leafwise closed. Indeed the foliation defined by θ_0 may have a leaf which admits no symplectic form. See Example 4.1 below. Similar examples can be obtained as applications of the ‘‘Taubes conjecture’’ theorem of Friedl-Vidussi[3], which says that a (closed 3-manifold) $\times S^1$ admits a symplectic structure if and only if it is a (surface bundle over S^1) $\times S^1$. On the other hand the leaves in $\{r < 1/2\}$ in Theorem 2.2 coil into the border leaf $\{r = 1/2\}$ (nearly) along a regular fibration $N^{2n-1} \rightarrow S^1$. The existence of ν guarantees that N^{2n-1} admits such a fibration. Also for other results in symplectic topology, it will be interesting to find their counterparts in leafwise symplectic foliation theory.

Example 4.1. Let S^4 be the 4-sphere $\{x_1^2 + \cdots + x_5^2 = 1\} \subset \mathbb{R}^5$, S^3 the equator $\{x_5 = 0\} \subset S^4$, and P the poles $\{x_5 = \pm 1\} \subset S^4$. Take an even bump function f which is supported in a small interval $[-\varepsilon, \varepsilon]$ and satisfies $\int_{-\varepsilon}^{\varepsilon} \{f(x)\}^2 dx = 2$. We define the 1-form θ_0 on $S^4 \times S^1$ ($s \in S^1$) by

$$\theta_0 = f(x_5)dx_5|_{TS^4} + \left(\int_0^{x_5} f(x)dx \right) ds.$$

Then $\gamma = f(x_5)ds$ satisfies $d\theta_0 = \theta_0 \wedge \gamma$. Take $\lambda = (x_1 dx_2 - x_2 dx_1 + x_3 dx_4 - x_4 dx_3)|_{S^3}$ on S^3 , pull it back under the meridian projection $\pi : S^4 \setminus P \rightarrow S^3$, and define the 1-form v_0 by $v_0 = (1 - x_5^2)\pi^*\lambda + x_5 ds$. Then $\omega = dv_0 + \gamma \wedge v_0$ satisfies $\theta_0 \wedge \omega^2 > 0$. However we know that the closed leaf $S^3 \times S^1$ of the foliation defined by θ_0 admits no symplectic structures.

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